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Intelligent Risk Processing and Opportunity Formation in Financial Markets: The Superior Performance of the HERC Algorithm in Efficient Portfolio Construction

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Abstract

Hierarchical portfolio optimization methods, particularly the Hierarchical Equal Risk Contribution (HERC) approach, have become increasingly prominent in financial research due to their effectiveness in balancing risk and enhancing diversification. Unlike traditional methods such as Equal-Weight (EW) and Inverse Volatility (IV), which rely on oversimplified assumptions and often underperform in volatile markets, HERC allocates capital by distributing risk more efficiently across assets. This study examines the performance of the HERC model relative to EW and IV to determine its ability to convert risk into investment opportunities under fluctuating market conditions. The methodology follows a structured process that includes deriving variables from multiple data sources, conducting thorough data cleaning and normalization, and implementing traditional allocation models as benchmarks. Advanced hierarchical clustering techniques are then applied to provide a more innovative allocation framework. Rigorous hypothesis testing is used to validate the results, and portfolio performance is evaluated using established statistical metrics. Findings reveal that HERC—especially its single linkage and average linkage versions—delivers substantially higher risk-adjusted returns, as measured by the Sharpe and Sortino ratios, compared to EW and IV. The proposed methodology not only improves overall investment outcomes but also enables more effective risk and return management, making it a strong alternative to conventional portfolio construction and risk evaluation approaches.

Keywords

Asset allocation, Hierarchical equal risk contribution (HERC), Risk management, Unsupervised learning.

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Introduction

In the volatile financial world, risk management has always been a significant challenge. Risk is traditionally viewed as a threat to returns; however, the ability to transform risk into opportunity is key to successful investing. The increasing integration of global financial markets necessitates advanced quantitative models (Rostami, et al 2025). In recent years, advanced algorithms have increasingly played a crucial role in enhancing investment management processes. Despite notable advancements in asset allocation algorithms, many traditional methods, such as the Markowitz model, still lack adequate efficiency in addressing real-world complexities due to their simplifying assumptions (Markowitz, 1952). Conversely, innovative approaches like the Hierarchical Equal Risk Contribution (HERC) algorithm have recently emerged, but comprehensive research comparing its performance with other advanced asset allocation models remains limited (Raffinot, 2018). Although significant theoretical and practical progress has been made in asset allocation, traditional models continue to face challenges such as complex asset correlations and imbalanced risk distribution in portfolios (Huang & Gao, 2021). These limitations not only lead to inefficient portfolios but also hinder the potential to improve risk-adjusted returns. The advent of new technologies, including machine learning in asset management, has created unparalleled opportunities for refining asset allocation methods (Schwendner et al., 2021; Menvouta et al., 2023). This research endeavor seeks to assess the efficacy of the Hierarchical Equal Risk Contribution (HERC) algorithm as an innovative method for risk management and asset distribution. The research seeks to assess HERC's performance compared to traditional and innovative models, identify its advantages and limitations, and provide recommendations for enhancing its efficiency in real market conditions. The HERC algorithm, introduced by Thomas Raffinot, represents a cutting-edge approach to portfolio risk management. By leveraging hierarchical clustering and emphasizing balanced risk distribution, HERC effectively manages risk while delivering more stable returns (Raffinot, 2018). As financial markets grow increasingly complex and economic crises emerge, investors seek tools that not only ensure sustainable returns but also guarantee balanced risk distribution (Millea & Edalat, 2022). The structure of this paper is organized as follows: the first section examines the theoretical foundations of the study, introducing the concept of hierarchical equal risk contribution portfolios and analyzing methodologies proposed by Lopez de Prado and Raffinot (Raffinot, 2017; López de Prado, 2016). Subsequent sections review the research background, discuss the dataset and associated challenges, and explain the research methodology. Ultimately, the findings derived from the empirical analyses will be presented and interpreted. This research endeavors to validate the HERC algorithm's ability to convert risk into opportunity and construct efficient portfolios. The findings are expected to open new avenues for investors and researchers in risk management and asset allocation while addressing existing gaps in the literature on asset allocation (Sen & Mehtab, 2021; Sajadi et al., 2024). The necessity of this study lies in addressing real market challenges and leveraging recent advancements in financial technology.

Theoretical Framework

Risk management is a process in which investors and financial managers identify, evaluate, and manage risks associated with assets and investments (Nourahmadi & Sadeqi, 2021). This concept has been thoroughly explored in both classical and modern financial theories, such as Markowitz's portfolio theory (Markowitz, 1952). Asset allocation involves distributing capital across various asset classes (e.g., stocks, bonds, and alternative assets) to optimize returns while managing risk. Within this domain, traditional models like Minimum Variance and advanced models like Hierarchical Equal Risk Contribution (HERC) hold particular significance (Raffinot, 2018). The HERC algorithm is a novel approach to portfolio management that leverages hierarchical clustering techniques to balance risk distribution among assets. By utilizing hierarchical structures, HERC addresses the limitations of traditional models such as Markowitz's and equitably distributes risks across the portfolio. Modern Portfolio Theory (MPT), introduced by Harry Markowitz, is founded on the risk-return hypothesis, proposing that portfolio optimization should maximize expected returns while minimizing risk. Additionally, models like Equal Risk Contribution (ERC) and parametric models aim to distribute risks evenly among assets (Maillard et al., 2010). However, these models often struggle with the complexities of asset correlations, paving the way for clustering-based methods like HERC. Hierarchical clustering methods, classified as unsupervised machine learning techniques, utilize asset correlation data to create tree-like structures for grouping assets (Duarte & De Castro, 2020). These structures, employed in the HERC algorithm, enable the identification of complex relationships among assets, thereby improving asset allocation efficiency and portfolio optimization. While traditional theories such as Markowitz's model and risk-balancing approaches like ERC hold a prominent place in financial literature, their limitations—such as the assumption of linear correlations and the inability to handle complex market structures—highlight the importance of algorithms like HERC. Despite the growing relevance of HERC in asset allocation, comprehensive studies analyzing its performance and comparing it with other models remain scarce, creating a critical gap in financial literature (Huang & Gao, 2021).

1. INVERSE VARIANCE PORTFOLIO

The logic behind the Inverse Variance (IV) portfolio allocation strategy is fundamentally straightforward: in the IV strategy, risk is measured by variance, and assets are weighted inversely proportional to their variances. Consequently, based on the standard deviation of returns, the IVP assigns the following weights to N assets (Ferretti, 2022):

$$w_{IV} = \frac{\frac{1}{\sigma_i^2}}{\sum_{i=1}^{N} \frac{1}{\sigma_i^2}} \tag{1}$$

where:

wi: Weight of asset i in the portfolio.

σ_i^2: Variance of the returns of asset i.

N: Total number of assets in the portfolio.

This method ensures that assets with lower variance receive higher weights, aligning the portfolio's construction with the principle of minimizing risk by favoring less volatile assets.

The primary advantage of this method lies in its computational simplicity. However, its accuracy and efficiency may diminish when faced with limitations and assumptions, such as the independence of assets or the exclusion of expected returns. Consequently, in practice, more advanced methods and models may be required to construct an optimal investment portfolio.

2. EQUAL-WEIGHT PORTFOLIO

In this straightforward method, each asset is assigned an equal weight. This approach is effective when the returns of all assets are entirely uncertain and random, with no subjective biases or preferences influencing the allocation.

$$w_i = \frac{1}{n} \tag{2}$$

The portfolio weight vector is defined as:

$$w_{EW} = \left(\frac{1}{n},\dots,\frac{1}{n}\right)^T \tag{3}$$

DeMiguel et al. (2009) made noteworthy findings, showing that despite its simplicity, the equal-weight portfolio often outperforms more complex strategies (DeMiguel et al., 2009). Specifically, the authors found that among 14 evaluated models, equal-weight portfolios consistently demonstrated superior performance in terms of Sharpe ratio, actual returns, and turnover in out-of-sample backtests. They argue that the benefits of diversification outweigh estimation errors, causing many sophisticated methods to underperform relative to this simple allocation strategy consistently.

3. HIERARCHICAL RISK PARITY

Hierarchical Risk Parity (HRP) is a portfolio optimization approach that is applied through a three-step process. The hierarchical clustering method is implemented using the Scipy library. Below is a description of each step in terms of its implementation stages.

Hierarchical Clustering

First, the correlation matrix is used to form clusters using the agglomerative hierarchical clustering algorithm. This ensures that assets within clusters are as similar as possible in terms of risk characteristics.

Sorting the Matrix

Second, the assets are sorted on the covariance matrix in such a way that similar assets are grouped closer together, facilitating a more balanced risk distribution across the portfolio.

Recursive Bisection

Third, in the recursive bisection stage, the sorted covariance matrix is used to allocate weights in such a way that the clusters achieve equal sizes. This process involves repeatedly halving the covariance matrix into sub-clusters until each asset is uniquely assigned to a cluster. To initiate this process, the algorithm starts with the following initialization.

Initialization of Clusters and Asset Weights:

Define the list of clusters as C0 = {C0} where C0 = {C0}n = 1,...,N i.e., all assets start in a single cluster.

Set the initial asset weights as $w_n = 1$, for all n∈[1,...,N]

Termination Condition:

If $|C_i| = 1$ for all Ci ∈ C , stop.

Cluster Division:

For each cluster Ci ∈ C, where, $|C_i| > 1$, continue.

Recursive Bisection:

Divide each cluster Ci into two subclusters Ci1 ∪ Ci2 , such that $|C_i^1| = \text{int}\left[\frac{1}{2}|C_i|\right]$.

Define the weights within each subcluster Cij as:

$$w_i^{(j)} = \frac{tr\left[\hat{\Sigma}_i^{(j)}\right]^{-1}}{\sum_i tr\left[\hat{\Sigma}_i^{(j)}\right] - 1} \quad for \quad j = 1,2 \tag{4}$$

where $\hat{\Sigma}_i^{(j)}$ is the covariance matrix for the subcluster C_{ij}.

Define and calculate the variance for each subcluster C_{ij} as:

$$V_i^j = w_i^{(j)\mathrm{T}} \hat{\Sigma}_i^{(j)} w_i^{(j)} \quad for \quad j = 1,2 \tag{5}$$

Compute the partition factor α_1 and α_2 as:

$$\alpha_1 = 1 - \frac{V_i^1}{V_i^1 + V_i^2} \ , \ \alpha_2 = 1 - \alpha_1 \tag{6}$$

Rescale the asset allocations w_n for all n ∈ Ci1 by factor α1.

Rescale the asset allocations w_n for all n ∈ Ci2by factor α2.

Repeat step 2 in a loop until the termination condition is met.

This description outlines the key steps in the Hierarchical Risk Parity (HRP) algorithm, which employs hierarchical clustering to balance risk across different assets and groups them recursively until each cluster contains a single asset. The process ensures that assets with higher correlation are grouped together and that risk is allocated in a balanced manner within each cluster (Deković & Šimović, 2025; Jain & Jain, 2019).

4. HIERARCHICAL EQUAL RISK CONTRIBUTION

The HERC method is a portfolio optimization technique based on hierarchical clustering and equal risk contribution across clusters of assets. It consists of a four-step process, which is similar to the Hierarchical Risk Parity (HRP) method but includes an additional step to determine the optimal number of clusters. It also differs in the assumptions regarding recursive halving. The four stages of the HERC method are as follows:

(1) Hierarchical Correlation Clustering

In this first step, a correlation matrix is transformed into a distance matrix. The distance matrix is then used to form clusters via a hierarchical agglomerative clustering algorithm. This process follows these steps:

a) Convert the Correlation Matrix:

The correlation matrix, which captures the relationship between asset returns, is converted into a distance matrix. The distance between two assets is generally computed as:

$$\text{Distance } (i, j) = 1 - corr(i, j) \quad (7)$$

b) Hierarchical Clustering:

Using the distance matrix, hierarchical clustering is applied. This technique starts with each asset as its own cluster and iteratively merges clusters based on their similarity (minimized distance) until all assets are in a single cluster.

c) Hierarchical Structure:

The resulting hierarchical structure or dendrogram provides a visual representation of asset relationships, which is crucial for the next steps.

(2) Determining the Optimal Number of Clusters

Determining the optimal number of clusters after hierarchical clustering is crucial, as it shapes the portfolio's structure by specifying sub-clusters. Methods like Silhouette Score, Gap Statistic, and Elbow Method are used to identify the ideal cluster count. This choice balances risk distribution depth with diversification, ensuring an effective allocation strategy.

(3) Recursive Hierarchical Halving

In this step, the sorted covariance matrix (from the clustering result) is recursively halved. The goal is to split the matrix into sub-clusters based on the hierarchical structure until the optimal number of clusters is achieved. This process includes:

a) Recursive Halving:

Starting with the covariance matrix, the assets are recursively divided into sub-clusters based on the hierarchical tree. This continues until each cluster has been divided into an optimal number of sub-clusters

b) Assigning Weights:

For each resulting sub-cluster, the weights are assigned to assets by balancing the risk contribution across all assets within that sub-cluster. The weights are calculated iteratively by considering the risk (variance) of each asset and its contribution to the overall cluster risk. The final weights are adjusted by the risk contribution factor, which ensures that the risk is equally distributed among all assets.

To implement the Hierarchical Equal Risk Contribution (HERC) algorithm, the process begins with initializing the algorithm and iteratively refining the asset weights through a recursive halving and equal risk distribution procedure. Below is the step-by-step implementation.

Step (1): Initialization

1. Initialize Clusters:

Start by setting the list of items (assets) as C0 = {C0} with C0 = {n}, n = 1,…,N. Here, N is the total number of assets in the portfolio.

2. Set Initial Weights:

The initial weights w_n for each asset are set to 1:

$$w_n = 1, \forall n \in [1, \cdots, N] \tag{8}$$

This assumes an equal initial distribution of weights across all assets.

Step (2): Recursive Clustering and Halving Process

For each iteration i = 0 to k* - 1, where k* is a predetermined number of clusters:

3. Ensure the Cluster Has More Than One Asset:

The algorithm continues only for clusters Ci where Ci > 1, i.e., the cluster has more than one asset.

4. Split the Clusters:

Divide each cluster C_i into two sub-clusters: Ci1 (left sub-cluster) and Ci2 (right sub-cluster).

5. Assign Initial Weights for the Sub-clusters:

The weights inside each sub-cluster Ci(j) for j= 1,2 are assigned as follows:

$$w_i^{(j)} = \frac{tr\left[\hat{\Sigma}_i^{(j)}\right]^{-1}}{\sum_i tr\left[\hat{\Sigma}_i^{(j)}\right] - 1} \tag{9}$$

Here, $\hat{\Sigma}_i^{(j)}$ is the covariance matrix of sub-cluster Cij.

6. Calculate Variance for Each Sub-cluster:

The variance of each sub-cluster $V_i^{(j)}$ for j = 1.2 is computed as:

$$v_i^j = w_i^{(j)\mathrm{T}} \hat{\Sigma}_i^{(j)} w_i^{(j)} \tag{10}$$

7. Calculate the Split Factor:

The factor α1 for the left sub-cluster and α2 for the right sub-cluster are calculated using the risk contributions of each sub-cluster:

$$\alpha_1 = 1 - \frac{RC_i^1}{RC_i^1 + RC_i^2}, \quad \alpha_2 = 1 - \alpha_1 \tag{11}$$

where RCi1 and RCi2 are the risk contributions of the left and right sub-clusters, respectively.

8. Rescale Weights for Each Sub-cluster:

Rescale the weights inside each sub-cluster:

$$\begin{aligned} &w_n \quad \forall n \in C_i^1 \quad by \quad \alpha_1 \\ &w_n \quad \forall n \in C_i^2 \quad by \quad \alpha_2 \end{aligned} \tag{12}$$

Step (3): Cluster Weighting

Once the clusters have been divided and the weights adjusted, the final weights between and within clusters are computed.

1. Within-Cluster Weights:

The weights within each cluster are calculated using an inverse variance allocation:

$$w_i^* = \frac{tr\left[\hat{\Sigma}_i\right]^{-1}}{\sum_i tr\left[\hat{\Sigma}_i\right] - 1} \tag{13}$$

where $\hat{\Sigma}_i$ represents the covariance matrix for each cluster C_i.

2. Final Asset Weights:

The final asset weights are obtained by multiplying the weights between clusters with the weights within clusters:

$$w_{HERC} = W.w \quad (14)$$

where:

W is the vector of weights between clusters, and w is the vector of weights within each cluster. The result ensures that the total portfolio weight sums to 1 (Raffinot, 2018; Deković & Šimović, 2025; Sjöstrand et al., 2020).

5. TYPES OF LINKAGE METHODS IN HIERARCHICAL CLUSTERING

A crucial consideration in hierarchical clustering is the selection of the distance metric and linkage method. The distance metric determines how the distance between two data points is measured, while the linkage method defines how the distances between clusters are combined to calculate the overall distance between them. Commonly used distance metrics include Euclidean distance, Manhattan distance, and cosine similarity, while common linkage methods include complete linkage, single linkage, and average linkage (Zhao & Karypis, 2002).

Hierarchical clustering is one of the widely used methods in data analysis aimed at identifying natural groupings (clusters) within data. This method reveals the underlying structure of data by creating a hierarchy of clusters. Among the various hierarchical clustering methods, Single Link, Average Link, and Ward's method are particularly popular due to their simplicity and effectiveness. In this research, we focus on the Single Link, Average Link, and Ward's methods. These methods form clusters using different distance metrics, each with its own advantages and disadvantages.

1. Single Link Method

In the Single Link method, the distance between two clusters is defined as the minimum distance between the nearest pair of points from the two clusters. In other words, at each step, the two clusters with the closest points are merged. This method is less sensitive to the shape of the clusters and can identify clusters with irregular shapes. However, it is prone to forming chain-like clusters, which may lead to counterintuitive results.

2. Average Link Method

In the Average Link method, the distance between two clusters is defined as the average distance between all pairs of points from the two clusters. This method is less prone to creating chain-like clusters compared to the Single Link method and tends to produce more spherical clusters. However, its sensitivity to the size of the clusters is a limitation.

3. Ward's Method

Ward's method aims to minimize the within-cluster variance and forms clusters by merging the two clusters that result in the smallest increase in the total within-cluster sum of squares (WCSS). This method generally produces clusters that are nearly equal in size and spherical in shape. However, for data with complex cluster shapes, the results may not be optimal (Dogan & Birant, 2022).

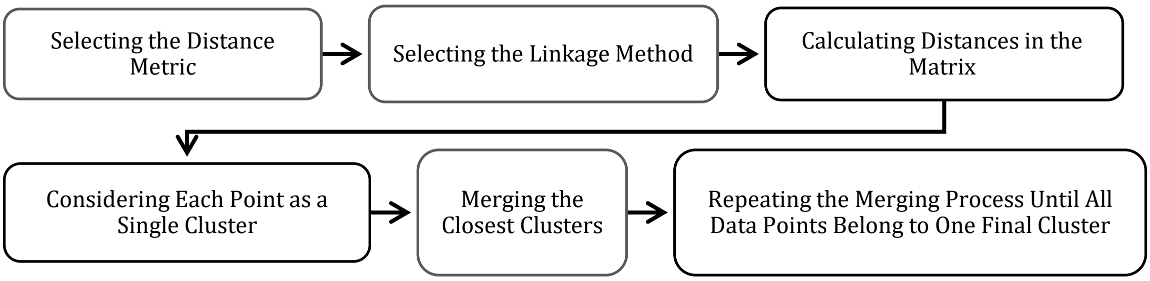
Table 1.
The methods of hierarchical clustering

Method	Distance Metric	Advantages	Disadvantages
Single Linkage	Minimum distance between closest points	Low sensitivity to cluster shape	Prone to chain-like clusters
Average Linkage	Average distance between all point pairs	Less prone to chain-like clusters, spherical clusters	Sensitive to cluster size
Ward's Method	Increase in WCSS upon merging clusters	Produces compact, spherical clusters of nearly equal size	Sensitive to complex cluster shapes

(Source: The Researcher's Findings)

In Fig. 1, the steps of the hierarchical clustering algorithm, considering the linkage method, are shown step by step. This algorithm is capable of uncovering the hierarchical structure of the data and progressively merging clusters so that one can observe and analyze the clusters more comprehensively at each stage (Asawa, 2021).

Figure 1.
Hierarchical Clustering Model Framework.



(Source: The Researcher's Findings)

6. PERFORMANCE EVALUATION METRICS

6.1 Sharpe Ratio

The Sharpe ratio, also known as the reward-to-volatility ratio, is a performance evaluation metric that measures the excess return over the risk-free rate, relative to the risk assumed by the investor (Sharpe, 1966). This metric was introduced by William Sharpe and is used to compare different investment portfolios. Generally, the Sharpe ratio is used to measure the risk of a portfolio, based on its total return (portfolio return) and the return of a risk-free investment (typically the return on government treasury bills). The Sharpe ratio shows how much additional return a portfolio has earned for each unit of risk. In comparing investment portfolios, the portfolio with a higher Sharpe ratio is usually considered superior because it indicates higher returns relative to the risk taken (Lim & Ong, 2020; Ferri, 2010).

6.2 Sortino Ratio

The Sortino ratio focuses on the downside risk of an investment portfolio and serves as an alternative to the standard deviation in the Sharpe ratio. This ratio measures the excess return for each unit of downside risk and is calculated by dividing the difference between the portfolio return and the risk-free rate by the downside deviation of the portfolio, which reflects the amount of downside risk (Venugopal & Sophia, 2020).

6.3 Maximum Drawdown

Maximum Drawdown (MDD) effectively presents the worst-case scenario for investors

(Raffinot, 2017). This metric answers the question of how much loss an investor might incur if they buy at the highest price and sell at the lowest possible price. Since this indicator measures the largest single drop from the peak to the lowest value of a portfolio's value, it is considered a downside risk metric. The Maximum Drawdown is expressed as a percentage and indicates how much of a portfolio's value has been lost relative to its highest point over a given time period. A portfolio with a lower maximum drawdown may be considered less risky and more stable. This metric shows how much an investment portfolio has lost in value during a market downturn. Generally, portfolios with lower maximum drawdowns are considered less risky during market declines because their value reduction during crisis times is relatively smaller.

6.4 Value at Risk

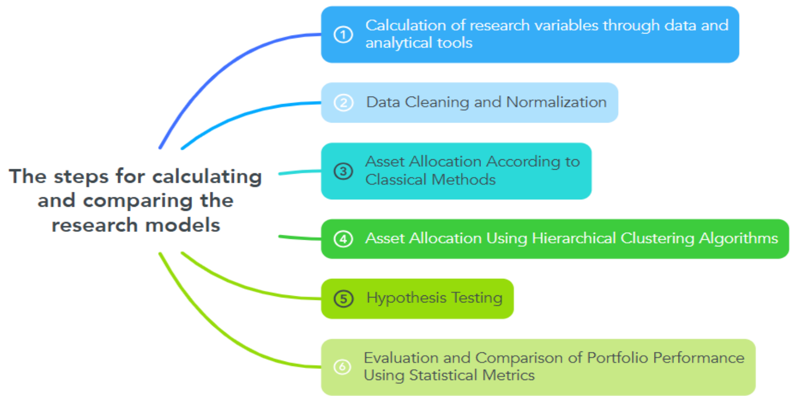
Value at Risk (VaR) is an important metric for assessing investment risk. It shows the probability that an investment might lose a certain amount of its value within a specified time period. Typically, VaR is determined at a specific confidence level, often expressed as a percentage. By calculating and monitoring VaR, investors can identify potential risks and make informed decisions about managing risk and adjusting their investment portfolios. This metric is popular among treasury managers, portfolio managers, and financial institutions, as it helps to assess how bad a situation could become. However, VaR has limitations and weaknesses and should be combined with other risk assessment tools and metrics for a more comprehensive view of investment risk (Seyfi et al., 2021).

6.5 Expected Shortfall

While VaR answers the question, "How bad can the situation get?" the Expected Shortfall (ES) asks, "What is the expected loss if the bad situation occurs?" In fact, Expected Shortfall builds upon Value at Risk. This metric is a function of the time horizon in days and the level of confidence. The Expected Shortfall is the average loss that exceeds the VaR threshold over a period of N days, assuming the loss exceeds the Xth percentile (the confidence level) of the loss distribution. This metric encourages diversification (Hallin & Trucíos, 2023).

In order to compare between different clustering methods and classical approaches, the steps are executed as shown in the figure below:

Figure 2.
Steps for Calculation and Comparison of Research Models.



(Source: The Researcher's Findings)

Literature Review

Past With the development of sophisticated algorithms that provide improved risk management techniques, the goal of creating an efficient portfolio has become a central focus of financial engineering research. One innovative approach in this domain is the Hierarchical Equal Risk Contribution (HERC) algorithm, which has gained traction for its ability to optimize portfolios by ensuring balanced risk distribution among assets. This literature review synthesizes existing research on the HERC algorithm and its application in constructing efficient portfolios. Markowitz's groundbreaking work, which focused on the trade-off between return and risk, established contemporary portfolio theory and served as the basis for portfolio optimization (Markowitz, 1952). Markowitz advocated for diversification to optimize asset allocation, a principle that still underpins many contemporary approaches. However, as financial markets have evolved, traditional methods have often been criticized for their oversimplified assumptions about asset correlations and risk distribution (Nourahmadi & Sadeqi, 2022). The application of data-driven and machine-learning-based recommender systems has demonstrated promising results in assisting investors to outperform the market by uncovering latent relationships among stocks (Nourahmadi, Rahimi, & Sadeqi, 2024). This calls for investigating alternate approaches that can more successfully deal with these constraints. The HERC algorithm, initially proposed by Raffinot (2018), leverages a hierarchical clustering framework to distribute risk equally among assets in a portfolio. This innovative approach addresses the problem of risk concentration, which is common in traditional portfolio constructions. By employing a clustering strategy, the HERC algorithm effectively identifies asset groupings that enhance diversification and stabilize returns (Duarte & De Castro, 2020). Huang (2021) and other recent researchers have assessed the HERC algorithm's performance across a range of marketplaces, proving its effectiveness in settings like the Chinese stock market. Huang's findings support the argument that HERC portfolios can yield better risk-adjusted returns compared to traditional models, thereby solidifying the importance of this algorithm in contemporary portfolio management (Huang & Gao, 2021). Furthermore, researching hierarchical clustering methods has highlighted their potential in improving the robustness of financial models under volatile market conditions (Nourahmadi & Sadeqi, 2023, Nourahmadi, et al, 2021). In conjunction with HERC, adaptive methods like adaptive seriational risk parity have emerged, which utilize machine learning techniques to optimize portfolio construction dynamically. The extensions of these adaptive techniques were examined by (Schwendner et al., 2021), who demonstrated how incorporating machine learning might improve risk parity strategies' performance in intricate financial contexts. These advancements indicate a shifting paradigm where machine learning tools are being harnessed alongside traditional financial theories to create more proactive portfolio management solutions. The interplay of hierarchical clustering and risk parity has also been emphasized in the works of (Millea & Edalat, 2022) and (Deković & Šimović, 2025). Millea and Edalat specifically focus on using deep reinforcement learning in conjunction with hierarchical risk parity, further illustrating the innovative combinations of techniques that can yield optimal portfolio compositions. Similarly, Deković and Šimović's research provides insights into the efficient implementation of hierarchical risk strategies in real-world settings, underscoring the significant implications for practitioners and portfolio managers. Moreover, research conducted by Sen et al (Sen & Mehtab, 2021; Sen & Dutta, 2023) on risk-based portfolio optimization in specific market sectors corroborates the relevance of HERC by suggesting that effective risk distribution not only enhances stability but also tailors investment strategies to better fit market dynamics. Their comparative studies highlight the necessity of incorporating advanced algorithms like HERC to develop portfolios that can adapt to the unique behaviors of various asset classes. The application of clustering methods in finance has further been explored by Cajas (Cajas, 2023), who employed graph theory to facilitate portfolio optimization. This research aligns with the findings of Nourahmadi & Sadeqi (Nourahmadi & Sadeqi, 2021), reinforcing the idea that clustering techniques can play a vital role in enhancing portfolio diversification processes. The synergistic effects of these methodologies open doors for further exploration into how these algorithms can be integrated to improve investment outcomes significantly. In summary, the literature indicates a growing consensus on the effectiveness of the hierarchical equal risk contribution algorithm as a pivotal tool in achieving portfolio efficiency. Its capacity to balance risk distribution among assets, coupled with the utilization of modern technologies such as machine learning and clustering techniques, illustrates the evolution of risk management in portfolio construction. As markets become increasingly complex, embracing methodologies like HERC will be crucial in developing strategies that not only safeguard against inherent financial risks but also aim for optimal returns across diversified portfolios. Future research should continue to investigate the integration of HERC with other computational techniques to further enhance portfolio performance and address the nuances of various market conditions.

Research Hypotheses

Hypothesis 1: The performance of the Hierarchical Equal Risk Contribution (HERC) algorithm is superior to that of the Equal Weight (EW) portfolio model.

Hypothesis 2: The performance of the Hierarchical Risk Parity (HRP) algorithm is superior to that of the Inverse Variance (IV) model.

Methodology

This study adopts a quantitative approach and employs numerical and statistical methods to identify and evaluate the optimal model. From an applied perspective, the objective is to develop a stock asset allocation framework that maximizes returns while minimizing risk, thereby providing practical insights for asset managers as well as individual and institutional investors. From a methodological standpoint, the research is conducted within a positivist paradigm and follows a deductive approach, whereby hypotheses derived from the existing theoretical literature on portfolio optimization and asset allocation algorithms are empirically tested using capital market data. The research strategy is based on correlation analysis and the comparison of models under identical conditions, with the aim of assessing the relative performance of different asset allocation methods. Data collection combines library research with the extraction of real market data, which form the basis for the quantitative analyses and statistical tests employed in the study. Initially, the adjusted stock prices of the sample (accounting for dividend payments and capital increases) over a ten-year period were extracted. After normalizing and cleansing the data, the second stage involved calculating classical models, including the Hierarchical Equal Risk Contribution (HERC) model, Hierarchical Risk Parity (HRP) algorithm, Inverse Variance (IV) model, and the Equal Weight (EW) portfolio, using Python software under the study's assumptions. In the third stage, the profitability and risk performance of these models were assessed using modern and post-modern portfolio metrics. Finally, the fourth stage was dedicated to comparing and evaluating the superiority of each strategy's performance through statistical tests.

Population and Sample

The statistical population comprises securities of all active issuers on the Tehran Stock Exchange (TSE) and Iran Farabourse market during the period from the beginning of 2013 to the end of 2022. The sample consists of daily data from 88 companies listed on the TSE that meet the following criteria:

1. The companies must have been listed on the TSE by the end of 2022.
2. Their stock symbols must have been active on the trading board throughout the study period with a complete price history.
3. The stocks must have been traded on at least 160 business days per year.

Sampling covered the period from 03/21/2013 to 03/18/2023, encompassing 2,408 trading days. Data were collected using Excel and subsequently processed, cleaned, and analyzed. Algorithms were executed, variables computed, and hypotheses tested using Python version 3.9.7 and libraries including NumPy, Pandas, Matplotlib, Seaborn, SciPy, Scikit-Learn, FinPy, and CVXPY. Statistical tests were performed in SPSS, while Excel was used for data visualization and chart creation.

Findings

Table 2 provides a summary of the descriptive statistics for the returns of the various portfolio strategies analyzed in the study. These statistics offer a comparative overview of the central tendency, dispersion, and risk-return characteristics of each strategy.

A. Statistical Analysis

The statistical analysis shows that the HERC single machine learning algorithm generally outperforms other machine learning models and classical methods such as EW and IV in terms of mean return. This indicates the potential of the HERC single algorithm to achieve higher returns. However, this algorithm also exhibits moderate volatility and higher kurtosis, suggesting heavier tails in the distribution of returns. HRP average has the highest standard deviation (0.199), indicating the most significant fluctuations. Both HRP single and HRP average demonstrate high kurtosis, reflecting heavy-tailed distributions and a higher likelihood of extreme positive or negative returns, representing higher risk for these strategies.

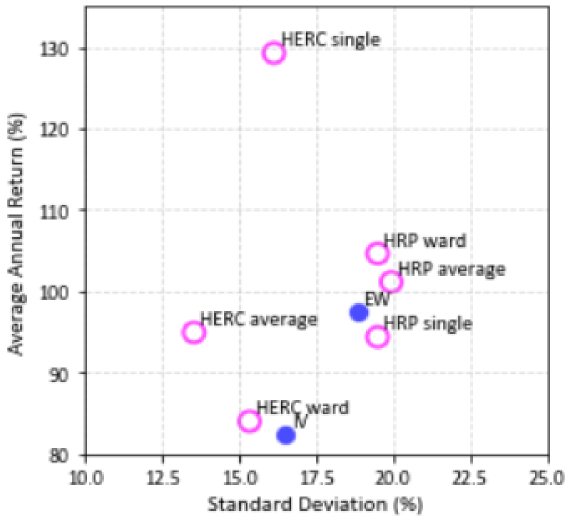
Table 2.
Central Tendency and Dispersion Indicators of Portfolio Returns

Algorithm	Mean	Standard Deviation	Min	25 Percentile	Median	75 Percentile	Max	Skewness	Kurtosis
HRP ward	1.046	0.195	-0.346	0.066	0.619	1.575	4.781	1.927	4.146
HRP single	0.944	0.195	-0.357	0.061	0.57	1.313	4.691	2.144	5.308
HRP average	1.012	0.199	-0.397	-0.009	0.569	1.441	5.241	2.165	5.336
HERC ward	0.84	0.154	-0.351	0.013	0.482	1.321	4.158	2.067	5.053
HERC single	1.293	0.162	-0.25	0.044	0.585	1.289	8.328	2.888	8.725
HERC average	0.949	0.136	-0.353	0.062	0.477	1.417	4.648	2.099	5.193
IV	0.824	0.165	-0.297	0.019	0.491	1.187	4.082	2.092	5.116
EW	0.976	0.188	-0.332	0.054	0.598	1.379	4.641	2.028	4.662

(Source: The Researcher's Findings)

In contrast, classical algorithms like EW and IV exhibit greater stability: EW, with a mean return of 0.976 and a standard deviation of 0.188, provides a reasonable balance between return and volatility. IV demonstrates better performance in adverse market conditions, with the smallest negative minimum return (-0.297), highlighting its resilience in critical scenarios.

Figure 3.
Return versus Risk Visualization for Different Portfolios.

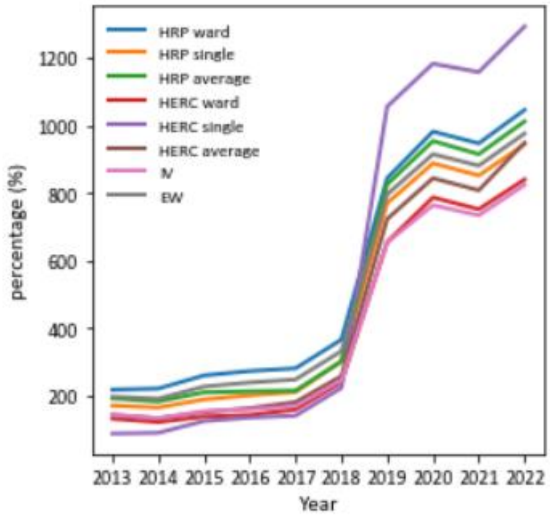


(Source: The Researcher's Findings)

The analysis of the chart indicates that the HERC single strategy achieves a better balance between return and risk compared to other methods. This approach allows investors to pursue their financial goals with greater confidence. Regarding the HERC methods, it is observed that their standard deviation is lower than that of the classical methods, EW and IV, which demonstrates the superiority of these methods over the two classical approaches, EW and IV.

Based on the data presented in Fig 4, the HERC single strategy significantly outperforms the other strategies, while the IV strategy recorded the lowest returns.

Figure 4.
Cumulative Return Trends of Different Portfolios.



(Source: The Researcher's Findings)

B. Risk-Based Performance Evaluation Variables

In this section, the performance of each classic and machine learning portfolio has been evaluated using the metrics presented in Table 3, which include maximum drawdown, annual VaR (5%), annual CVaR (5%), Sharpe ratio, and Sortino ratio. It highlights the superior performance of the HERC Single strategy in terms of risk-adjusted returns, evidenced by its high Sharpe and Sortino ratios.

Table 3.
Average Risk-Based Performance Metrics of Classical and Machine Learning Portfolios

Algorithm	MD	VaR	CVaR	Sharpe	Sortino
Machine Learning					
HRP Ward	25.5%	29.8%	37.1%	2.38	4.31
HRP Single	26.5%	30.9%	38.3%	2.13	3.78
HRP Average	27.9%	30.8%	39.2%	2.14	3.84
HERC Ward	20.7%	23.1%	30.5%	2.01	3.67
HERC Single	19.4%	23.0%	31.2%	4.62	6.89
HERC Average	17.4%	19.5%	25.7%	2.74	5.38
Classical					
IV	22.4%	25.8%	32.88%	2.14	3.84
EW	25.4%	29.1%	36.82%	1.98	3.75

(Source: The Researcher's Findings)

In terms of Maximum Drawdown, machine learning algorithms, particularly HRP Average with a drawdown of 27.96%, exhibit the highest maximum loss, indicating an increased likelihood of negative returns during critical market conditions. Other machine learning algorithms, such as HRP Single and HRP Ward, also experience notable drawdowns. Conversely, machine learning models like HERC demonstrate lower drawdowns, reflecting more stable performance and reduced risk, especially in bearish market scenarios.

For VaR at 5% (Value at Risk at 5%), HRP Single has the highest value at 30.93%, signifying higher risk levels for this model. Other machine learning models, such as HRP Average and HRP Ward, also show varying levels of risk exposure.

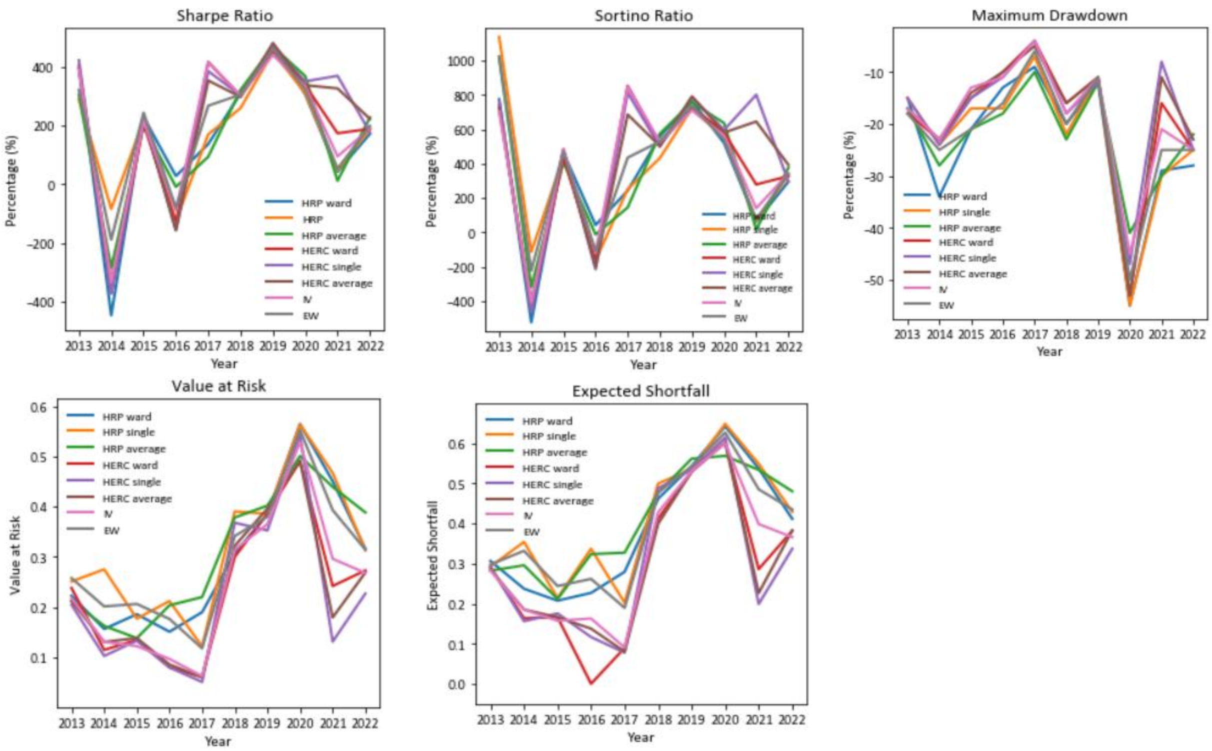
Regarding CVaR at 5% (Conditional Value at Risk at 5%), HRP Average records the highest value at 39.24% among machine learning algorithms, indicating a higher risk tolerance. Classical models such as EW (36.82%) and IV (32.88%) exhibit greater risk compared to the HERC machine learning models.

In terms of the Mean Sharpe Ratio, machine learning algorithms, including HERC Single (4.62) and HERC Average (2.74), achieve the highest Sharpe ratios, indicating favorable returns relative to risk. Classical models such as IV (2.14) also perform reasonably well, but machine learning algorithms outperform them on this metric. Finally, for the Mean Sortino Ratio, HERC Single (6.89) and HERC Average (5.38) demonstrate the highest values, reflecting their superior ability to manage downside risk. Classical models like IV (3.84) and EW (3.75) offer similar results for this ratio, but machine learning algorithms, particularly HERC Single, show significantly better performance in this aspect.

In conclusion, while machine learning algorithms provide varying results on risk measures such as VaR and CVaR, they achieve higher Sharpe and Sortino ratios compared to classical models. Classical models like EW and IV are more stable and exhibit lower risk, but in terms of risk-adjusted returns, machine learning algorithms, especially HERC Single and HERC Average, demonstrate superior performance.

Figure 5.

Performance metrics of various portfolios from 21-3-2013 to 19-3-2023 - from top left to bottom: Sharpe ratio, Sortino ratio, Maximum Drawdown, Value at Risk, Expected Shortfall.



(Source: The Researcher's Findings)

C. Hypothesis Testing of the Research

The Kruskal-Wallis test is a non-parametric statistical method used to compare the rankings of different groups, particularly when there are two or more groups involved. This test provides two main outputs (Safavi Iranji et al., 2024).

1. Rankings of Groups:
In the first output, groups are ranked based on the variables under consideration. Higher rankings indicate better performance of the method for a given metric, while lower rankings suggest weaker performance for the same metric.

2. Test Statistics and Significance Levels:
The second output reports the Kruskal-Wallis test statistics and their corresponding significance levels. In this study, hypotheses are considered supported if the significance level is below 10%.

There is a direct relationship between higher values and rankings of metrics like the Sharpe and Sortino ratios and the improved performance of models. In other words, models achieving higher values and rankings in these metrics demonstrate superior risk-adjusted returns and a better return-to-volatility ratio.

Conversely, an inverse relationship is observed between lower values and rankings of metrics like Maximum Drawdown, Value at Risk (VaR), and Conditional Value at Risk (CVaR) with better model performance. This means that models with lower values and rankings in these criteria exhibit greater stability and reduced risk of loss.

The results of the Kruskal-Wallis test rankings, based on the five selected criteria and the clustering linkage methods, are presented in Table 4.

Table 4.
Kruskal-Wallis Statistical Test Results

Metric	Mean Rank		KW	Mean Rank		KW	Mean Rank		KW
	EW	HERC single		EW	HERC average		EW	HERC ward	
Sharpe Ratio	7.8	13.4	3.221**	9.7	11.3	0.366	10.4	10.6	0.006
Sortino Ratio	7.85	13.55	3.014**	9.5	11.5	0.571	10.6	10.4	0.006
Maximum Drawdown	12.5	8.5	2.286	12.5	8.5	2.286	11.9	9.1	1.12
Conditional VaR (CVaR)	11.6	9.4	0.691	13	8	3.571**	11.9	9.1	1.12
Value at Risk (VaR)	12.1	8.85	1.557	13	8	3.571**	12	9	1.286
Metric	**IV**	**HRP single**	**KW**	**IV**	**HRP average**	**KW**	**IV**	**HRP ward**	**KW**
Sharpe Ratio	10.6	10.4	0.006	10.8	10.2	0.051	10.2	10.8	0.051
Sortino Ratio	10.4	10.6	0.006	10.7	10.3	0.023	10.2	10.8	0.051
Maximum Drawdown	9.5	11.5	0.571	9.2	11.8	0.966	9.7	11.3	0.366
Conditional VaR (CVaR)	9	12	1.286	8.8	12.2	1.651	9.4	11.6	0.691
Value at Risk (VaR)	9	12	1.286	9	12	1.286	9.1	11.9	1.12

Notes:
KW indicates the Kruskal-Wallis test statistic.
** indicate significance at the 10% level.
(Source: The Researcher's Findings)

The analysis of the Kruskal-Wallis test results indicates the following:

Sharpe Ratio: A significant difference at the 10% level was found between EW and HERC Single, indicating the superior performance of HERC Single in this metric. However, no significant differences were observed between EW and HERC Average, as well as EW and HERC Ward, suggesting that the performance of these methods is similar in this metric. Sortino Ratio: A significant difference at the 10% level was observed between EW and HERC Single, with HERC Single performing better. However, when comparing EW with HERC Average and EW with HERC Ward, no significant difference was found, indicating comparable performance between these methods. Maximum Drawdown: No significant differences were found between EW and the HERC models (Single, Average, Ward) in terms of maximum drawdown. This suggests that these models exhibit similar performance in minimizing losses during market downturns. Conditional Value at Risk (CVaR): No significant difference was found between EW and HERC Single. However, HERC Average showed better performance than EW at the 10% level, while no significant difference was reported between EW and HERC Ward. Value at Risk (VaR): No significant

differences were found in comparisons of EW with the HERC Single and HERC Ward models. The IV and HRP models (Single, Average, Ward) generally showed similar performance across most metrics. Although in some cases, such as with the Sharpe and Sortino ratios, the KW value approached significance, no statistical significance was reported at the 0.1 level.

The results suggest that HERC strategies outperform traditional strategies such as EW and IV in certain metrics, particularly in risk-adjusted return metrics like the Sharpe and Sortino ratios. These findings indicate that using data-driven and machine learning-based models can effectively improve portfolio management and lead to higher returns.

Discussion and Conclusion

The results of this research indicate that the Hierarchical Equal Risk Contribution (HERC) algorithm, as an innovative tool, shows remarkable potential in portfolio management and transforming risk into opportunity in financial markets. The HERC models, especially the single and average versions, exhibited superior performance in risk-adjusted return metrics, such as the Sharpe ratio and Sortino ratio, compared to traditional methods like Equal Weighting (EW) and Inverse Variance (IV). These findings demonstrate the ability of HERC to create an efficient balance between returns and risk management.

At the same time, the results showed that there was no significant difference in the maximum drawdown metric between the methods. These results, along with the superior performance of HERC, suggest that the algorithm is capable of maintaining stability and providing a conservative approach to risk management. Therefore, HERC can be considered as an optimal option for investors seeking to reduce risk and enhance returns. By leveraging clustering structures and analyzing nonlinear relationships between assets, HERC has secured a unique position among modern portfolio management methods. Additionally, by integrating hierarchical approaches and risk analysis, this algorithm goes beyond traditional methods and provides potential capacities for improving investment decision-making. However, to fully exploit this method, it is recommended that its performance be evaluated under various market conditions and economic cycles.

The findings of this research align with previous studies, such as the work of Vito Ciciretti (Ciciretti & Bucci, 2023), which showed that the use of clustering methods for portfolio optimization outperforms classical approaches. Furthermore, external studies, such as the research by Thomas Raffinot (2018), also confirm the results of this study. Raffinot's research showed that HERC-based portfolios, using a hierarchical approach and equal risk contribution allocation, performed better than other methods in metrics such as Conditional Drawdown at Risk (CDaR). Additionally, the findings of Lopez de Prado (2016) demonstrated that the HRP method (the parent approach of HERC) resolves instability issues and focuses on the problems of Markowitz methods, producing more diversified portfolios with lower risk.

Despite the positive findings, some studies, like Jain and Jain (2019), have shown that hierarchical algorithms, including HERC and HRP, are less sensitive to incorrect covariance estimates compared to methods like minimum variance or maximum diversification. This emphasizes the importance of accurate input parameter estimation to fully optimize the performance of HERC. To complement these findings and take further advantage of HERC's potential, it is recommended that the algorithm's performance be evaluated in various market conditions, as assessing its compatibility in volatile periods and different economic cycles can provide a better understanding of its stability. Additionally, combining HERC with classical methods could create a hybrid approach that leverages the strengths of both data-driven and traditional methods, leading to improved portfolio performance. Moreover, refining and improving input parameters and analyzing the impact of nonlinear relationships in financial data could contribute to further development of this algorithm.

Finally, this study, by highlighting the advantages of the HERC algorithm, demonstrates that this method can bring a significant transformation to portfolio management and serve as an efficient alternative to traditional methods. HERC not only offers higher returns but also, with its innovative risk management approach, provides investors with a powerful tool to turn risk challenges into investment opportunities.

Abbreviations

HERC ward	Hierarchical Equal Risk Contribution using Ward's Method Linkage
HERC single	Hierarchical Equal Risk Contribution Single Method Linkage
HERC average	Hierarchical Equal Risk Contribution using Average Method Linkage
HRP ward	Hierarchical Risk Parity using Ward's Method Linkage
HRP single	Hierarchical Risk Parity using Single Method Linkage
HRP average	Hierarchical Risk Parity using Average Method Linkage
IV	Inverse Variance
EW	Equal Weight Portfolio
Sharpe	Sharpe Ratio
Sortino	Sortino Ratio
MD	Maximum Drawdown
VaR	Value at Risk at 5.0%
CVaR	Conditional Value at Risk
KW	Kruskal-Wallis test statistic

REFERENCE

Asawa, Y. (2021). Modern machine learning solutions for portfolio selection. IEEE Engineering Management Review, 50(1), 94–112. https://ieeexplore.ieee.org/abstract/document/9627810.

Cajas, D. (2023). A graph theory approach to portfolio optimization. SSRN Working Paper. https://dx.doi.org/10.2139/ssrn.4602019.

Ciciretti, V., & Bucci, A. (2023). Building optimal regime-switching portfolios. Economic Modelling and Finance, 64, S1062940822001723. https://doi.org/10.1016/j.najef.2022.101837.

Deković, D., & Šimović, P. P. (2025). Hierarchical risk parity: Efficient implementation and real world analysis. Future Generation Computer Systems, 107, 107744. https://doi.org/10.1016/j.future.2025.107744.

DeMiguel, V., Garlappi, L., & Uppal, R. (2009). Optimal versus naive diversification: How inefficient is the 1/N portfolio strategy? Review of Financial Studies, 22(5), 1915–1953. https://doi.org/10.1093/rfs/hhm075.

Dogan, A., & Birant, D. (2022). K-centroid link: A novel hierarchical clustering linkage method. Applied Intelligence, 52, 1–24. https://doi.org/10.1007/s10489-021-02624-8.

Duarte, F. G., & De Castro, L. N. (2020). A framework to perform asset allocation based on partitional clustering. IEEE Access, 8, 110775–110788. https://doi.org/10.1109/access.2020.3001944.

Ferretti, S. (2022). On the modeling and simulation of portfolio allocation schemes: An approach based on network community detection. Computational Economics, 62(3), 969–1005. https://link.springer.com/article/10.1007/s10614-022-10288-w.

Ferri, R. (2010). All about asset allocation (2nd ed.). McGraw-Hill Education. https://search.worldcat.org/title/663882714.

Hallin, M., & Trucíos, C. (2023). Forecasting value-at-risk and expected shortfall in large portfolios: A general dynamic factor model approach. Econometrics and Statistics, 27, 1–5. https://doi.org/10.1016/j.ecosta.2021.04.006.

Huang, W., & Gao, X. (2021). Evaluating hierarchical equal risk contribution portfolios in the Chinese stock market. Journal of Mathematical Finance, 12(1), 179–195. https://doi.org/10.4236/jmf.2022.121011.

Jain, P., & Jain, S. (2019). Can machine learning-based portfolios outperform traditional risk-based portfolios? The need to account for covariance misspecification. Risks, 7(3), 74. https://doi.org/10.3390/risks7030074.

Lim, T., & Ong, C. (2020). Portfolio diversification using shape-based clustering. Journal of Financial Data Science, 3(1), 111. https://doi.org/10.3905/jfds.2020.1.054.

López de Prado, M. (2016). Building diversified portfolios that outperform out of sample. Journal of Portfolio Management, 42(4), 59–69. https://doi.org/10.3905/jpm.2016.42.4.059.

Maillard, S., Roncalli, T., & Teïletche, J. (2010). The properties of equally weighted risk contribution portfolios. Journal of Portfolio Management, 36(4), 60–70. https://doi.org/10.3905/jpm.2010.36.4.060.

Markowitz, H. M. (1952). Portfolio selection. Journal of Finance, 7(1), 77–91. https://doi.org/10.2307/2975974.

Menvouta, E. J., Serneels, S., & Verdonck, T. (2023). Portfolio optimization using cellwise robust association measures and clustering methods with application to highly volatile markets.

Journal of Financial Data Science, 9, 100097. https://doi.org/10.1016/j.jfds.2023.100097.

Millea, A., & Edalat, A. (2022). Using deep reinforcement learning with hierarchical risk parity for portfolio optimization. International Journal of Financial Studies, 11(1), 10. https://doi.org/10.3390/ijfs11010010.

Nourahmadi, M., & Sadeqi, H. (2021). Hierarchical risk parity as an alternative to conventional methods of portfolio optimization: A study of Tehran Stock Exchange. Iranian Journal of Finance, 5(4), 1–24. https://doi.org/10.30699/ijf.2021.289848.1242.

Nourahmadi, M., & Sadeqi, H. (2022). A machine learning-based hierarchical risk parity approach: A case study of a portfolio consisting of stocks of the top 30 companies on the Tehran Stock Exchange. Financial Research Journal, 24(2), 236–256. https://doi.org/10.22059/frj.2021.319092.1007146.

Nourahmadi, M., & Sadeqi, H. (2023). Portfolio diversification based on clustering analysis. Iranian Journal of Accounting, Auditing and Finance, 7(3), 1–16. https://doi.org/10.22067/ijaaf.2023.43078.1092.

Nourahmadi, Marziyeh, rasti, Fatemeh, & SADEGHI, HOJATOLLAH. (2021). A Review of Research on Financial Time Series Clustering: A Bibliometrics Approach. ADVANCES IS FINANCE AND INVESTMENT, 2(2), 23-57. SID. https://sid.ir/paper/391770/en

Nourahmadi, M. , Rahimi, A. and Sadeqi, H. (2024). Designing a Stock Recommender System Using the Collaborative Filtering Algorithm for the Tehran Stock Exchange. *Financial Research Journal*, *26*(2), 318-346. doi: 10.22059/frj.2023.360955.1007479

Raffinot, T. (2017). Hierarchical clustering-based asset allocation. Journal of Portfolio Management, 44(2), 89–102. https://doi.org/10.3905/jpm.2018.44.2.089.

Raffinot, T. (2018). The hierarchical equal risk contribution portfolio. SSRN Electronic Journal. https://dx.doi.org/10.2139/ssrn.3237540.

Rostami, M. , Rasti, F. and Abbasi, E. (2025). Copula-Based Risk Modeling: A Comparative Analysis of MCAViaR and Gaussian Copulas for Global Indices. *Journal of Mathematics and Modeling in Finance*, *5*(2), 77-106. doi: 10.22054/jmmf.2025.86227.1187

Safavi Iranji, M., Zanjirdar, M., Safa, M., & Jahangirnia, H. (2024). Asset allocation using nested clustered optimization algorithm: A novel approach to risk management in portfolio. Journal of Mathematical Modeling in Finance, 4(2), 137–157. https://doi.org/10.22054/jmmf.2025.82388.1149.

Sajadi, S. M. A., Barak, S., & Fereydooni, A. (2024). Online portfolio selection using macroeconomic pattern matching. SSRN Electronic Journal. https://papers.ssrn.com/sol3/papers.cfm?abstract_id=4960922.

Schwendner, P., Papenbrock, J., Jaeger, M., & Krügel, S. (2021). Adaptive seriational risk parity and other extensions for heuristic portfolio construction using machine learning and graph theory. Journal of Financial Data Science, 3(4), 65–83. DOI: 10.3905/jfds.2021.1.078.

Sen, J., & Dutta, A. (2023). Portfolio optimization for the Indian stock market. In Encyclopedia of Data Science and Machine Learning (pp. 1904–1951). IGI Global. https://www.igi-global.com/chapter/portfolio-optimization-for-the-indian-stock-market/317595.

Sen, J., & Mehtab, S. (2021). A comparative study of optimum risk portfolio and eigen portfolio on the Indian stock market. International Journal of Business Forecasting and Marketing Intelligence, 7(2), 143–193. https://doi.org/10.1504/IJBFMI.2021.120155.

Seyfi, S. M., Sharifi, A., & Arian, H. (2021). Portfolio value-at-risk and expected shortfall using an efficient simulation approach based on Gaussian mixture model. Mathematics and

Computers in Simulation, 190, 1056–1079. https://doi.org/10.1016/j.matcom.2021.05.029.

Sharpe, W. F. (1966). Mutual fund performance. Journal of Business, 39(1), 119–138. https://www.jstor.org/stable/2351741?searchText=Mutual%20fund%20performance&searchUri=%2Faction%2FdoBasicSearch%3FQuery%3DMutual%2Bfund%2Bperformance%26so%3Drel&ab_segments=0%2Fbasic_search_gsv2%2Fcontrol&refreqid=fastly-default%3A32de7d5ea9a6a553b1e108b638c1f88f.

Sjöstrand, D., Behnejad, N., & Richter, M. (2020). Exploration of hierarchical clustering in long-only risk-based portfolio optimization (Doctoral dissertation). Copenhagen Business School, Copenhagen. https://research.cbs.dk/files/62178444/879726_Master_Thesis_Nima_Daniel_15736.pdf.

Venugopal, M., & Sophia, S. (2020). Examining Sharpe ratio, ASR, Sortino, Treynor and information ratio in Indian equity mutual funds during the pandemic. International Journal of Management, 11(11). https://papers.ssrn.com/sol3/papers.cfm?abstract_id=3780440.

Zhao, Y., & Karypis, G. (2002). Evaluation of hierarchical clustering algorithms for document datasets. In Proceedings of the International Conference on Information and Knowledge Management (pp. 515–524). https://doi.org/10.1145/584792.584877.